Giant Gravitons with NSNS B field

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ABSTRACT

We study the motion of a D(8-p)-brane probe in the background created by a stack of non-threshold (D(p-2), Dp) bound states for $2 \leq p \leq 6$. The brane probe and the branes of the background have two common directions. We show that for a particular value of the worldvolume gauge field there exist configurations of the probe brane which behave as massless particles and can be interpreted as gravitons blown up into a fuzzy sphere and a noncommutative plane. We check this behaviour by studying the motion and energy of the brane and by determining how supersymmetry is broken by the probe as it moves under the action of the background.

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1 Introduction

One of the most interesting things we have recently learnt from string theory is the fact that a system can increase its size with increasing momentum. There are several manifestations of this phenomenon, which is opposite to the standard field theory intuition, in several contexts related to string theory such as the infrared ultraviolet connection [1] or the noncommutative geometry [2].

In ref. [3] McGreevy, Susskind and Toumbas found another example of the growth in size with energy. These authors considered a massless particle moving in a spacetime of the form $AdS_m \times S^{p+2}$ and discovered that there exists a configuration in which an expanded brane (the giant graviton) has exactly the same quantum numbers as the point-like particle. This expanded brane wraps the spherical part of the spacetime and is stabilized against shrinking by the flux of the Ramond-Ramond (RR) gauge field. The size of the giant graviton increases with its angular momentum and, as the radius of the brane cannot be greater than the radius of the spacetime, one gets that there exists an upper bound for the momentum of the brane. This fact is a realization of the so-called stringy exclusion principle. Moreover, in refs. [4, 5] was proved that the giant gravitons of ref. [3] are BPS configurations which preserve the same supersymmetry as the point-like graviton. It was also shown in [4, 5] that there also exist gravitons expanded into the AdS part of the spacetime which, however, do not have an upper bound on their angular momentum due to the non-compact nature of the AdS spacetime.

The overall physical picture one gets from these results is that for high momenta the linearized approximation to supergravity breaks down and one is forced to introduce interactions in order to describe the dynamics of the massless modes of the theory. An effective procedure to represent these interactions is to assume that the massless particles polarize and become a brane. The precise mechanism of this polarization is the so-called Myers dielectric brane effect [6].

The blowing up of gravitons into branes can take place in backgrounds different from $AdS_m \times S^{p+2}$. Indeed, in ref. [7] it was found that there are giant graviton configurations of D(6-p)-branes moving in the near-horizon geometry of a dilatonic background created by a stack of Dp-branes. For other aspects of the expanded graviton solutions see refs. [8]-[12].

In this paper we will find giant graviton solutions for probes which move in the geometry created by a stack of non-threshold bound states of the type (D(p-2), Dp) for $2 \leq p \leq 6$ [13]. These backgrounds are 1/2 supersymmetric and have been considered recently [14] as supergravity duals of noncommutative gauge theories. They are characterized by the presence of a non-zero Kalb-Ramond field B, directed along the Dp-brane, from the Neveu-Schwarz sector of the superstring, together with the corresponding RR fields. We will place in this background a brane probe in such a way that it could capture both the RR flux (as in the Dp backgrounds) and the flux of the B field. This last requirement implies that we must extend our probe along two directions parallel to the background and, actually, our probe will be a D(8-p)-brane wrapped on an S^{6-p} sphere transverse to the background and extended along a (noncommutative) plane parallel to it. We will verify that, by switching on a particular worldvolume gauge field, one can find configurations of the D(8-p)-brane which behave as a massless particle. We will check that the energy of these giant graviton configurations is exactly the same as that of a massless particle moving in the metric of the (D(p-2), Dp) background. Generically, the brane falls into the center of the gravitational potential along a trajectory which will be determined. We will also study the supersymmetry projection introduced by the brane and we will show that it breaks completely the supersymmetry of the background, exactly in the same way as a wave which propagates with the velocity of the center of mass of the brane. Thus, also from the point of view of supersymmetry, the expanded brane mimics the massless particle.

The plan of this paper is the following. In section 2 we will start our analysis by describing the (D(p-2),Dp) background. Then, we will consider a brane probe and its action and energy on the background will be studied. At the end of this section the giant graviton configurations will be characterized and the corresponding equations of motion will be integrated. Section 3 is devoted to the analysis of the supersymmetry of the problem for the particular case of the (D1, D3) background. First of all, we shall obtain the form of the Killing spinors. Then, the κ-symmetry of the probe is studied and its associated supersymmetry projection is obtained. Details of the determination of the Killing spinors for the (D1, D3) background are given in appendix A. Finally, in section 4 we will summarize our results and discuss some possible extensions of our work.

2 Giant gravitons in (D(p-2),Dp) backgrounds

The supergravity background we will consider is the one generated by a stack of N non-threshold bound states of Dp and D(p-2) branes for $2 \le p \le 6$. The metric and dilaton (in the string frame) for such a background are [13]:

$$\begin{aligned} ds^2 &= f_p^{-1/2}\Big[\,-(\,dx^0\,)^2 + \cdots + (\,dx^{p-2}\,)^2 + \,h_p\left((\,dx^{p-1}\,)^2 + (\,dx^p\,)^2\right)\Big] + \\ &\quad + f_p^{1/2}\Big[\,dr^2 + r^2\,d\Omega^2_{8-p}\,\Big]\,, \\ e^{\tilde\phi_D} &= f_p^{\frac{3-p}{4}}\; h_p^{1/2}\,, \end{aligned} \tag{2.1}$$

where $d\Omega^2_{8-p}$ is the line element of a unit $(8-p)$ sphere, r is a radial coordinate parametrizing the distance to the brane bound state and $\tilde\phi_D = \phi_D - \phi_D(r\rightarrow\infty)$. The Dp-brane of the background extends along the directions $x^0\cdots x^p$, whereas the D(p-2)-brane component lies along $x^0\cdots x^{p-2}$. The functions f_p and h_p appearing in (2.1) are:

$$\begin{aligned} f_p &= 1 + \frac{R^{7-p}}{r^{7-p}}\,, \\ h_p^{-1} &= \sin^2\varphi\, f_p^{-1} + \cos^2\varphi\,, \end{aligned} \tag{2.2}$$

with φ being an angle which characterizes the degree of mixing of the Dp and D(p-2) branes in the bound state. The parameter R, which we will refer to as the radius, is given by:

$$R^{7-p}\,\cos\varphi = N\,g_s\,2^{5-p}\,\pi^{\frac{5-p}{2}}\,(\,\alpha'\,)^{\frac{7-p}{2}}\;\Gamma\Big(\frac{7-p}{2}\Big)\,, \tag{2.3}$$

where N is the number of branes of the stack, g_s is the string coupling constant ($g_s = e^{\phi_D(r\to\infty)}$) and α' is the Regge slope.

This solution is also endowed with a rank two NSNS field B directed along the $x^{p-1}x^p$ (noncommutative) plane:

$$B = \tan\varphi\, f_p^{-1}\, h_p\, dx^{p-1}\wedge dx^p\ , \tag{2.4}$$

and is charged under RR field strengths, $F^{(p)}$ and $F^{(p+2)}$. The components along the directions parallel to the bound state are:

$$\begin{aligned} F^{(p)}_{x^0,x^1,\cdots,x^{p-2},r} &= \sin\varphi\,\partial_r\, f_p^{-1}\ ,\\ F^{(p+2)}_{x^0,x^1,\cdots,x^p,r} &= \cos\varphi\, h_p\partial_r\, f_p^{-1}\ . \end{aligned} \tag{2.5}$$

It is understood that the $F^{(p)}$'s for $p\geq 5$ are the Hodge duals of those with $p\leq 5$, *i.e.* $F^{(p)} = {}^*F^{(10-p)}$ for $p\leq 5$. In particular this implies that $F^{(5)}$ is self-dual, as is well-known for the type IIB theory. It is clear from the above equations that for $\varphi=0$ the (D(p-2),Dp) solution reduces to the Dp-brane geometry whereas for $\varphi=\pi/2$ it becomes a D(p-2)-brane smeared along the $x^{p-1}x^p$ directions.

It will be convenient for our purposes to use a particular coordinate system [3] for the unit $8-p$ sphere S^{8-p}. In order to define these coordinates, let us realize S^{8-p} as the surface in $\mathbb{R}^{9-p}$ whose equation is $(\,z^1\,)^2 + \cdots + (\,z^{9-p}\,)^2 \,=\, 1$. We shall parametrize z^1 and z^2 by means of the coordinates ρ and ϕ as follows:

$$z^1 \,=\, \sqrt{1\,-\,\rho^2}\,\cos\phi\ , \qquad\qquad z^2 \,=\, \sqrt{1\,-\,\rho^2}\,\sin\phi\ , \tag{2.6}$$

where $0\leq\rho\leq 1$ and $0\leq\phi\leq 2\pi$. Clearly, on S^{8-p}, one has:

$$(\,z^1\,)^2 \,+\, (\,z^2\,)^2 \,=\, 1-\rho^2\ , \qquad\qquad (\,z^3\,)^2 \,+\cdots+\, (\,z^{9-p}\,)^2 \,=\, \rho^2\ . \tag{2.7}$$

After a simple calculation one can demonstrate that, in terms of (ρ,ϕ), the metric of S^{8-p} takes the form:

$$d\Omega^2_{8-p} \,=\, \frac{1}{1-\rho^2}\,d\rho^2 \,+\, (1\,-\,\rho^2\,)\,d\phi^2 \,+\, \rho^2\, d\Omega^2_{6-p}\ , \tag{2.8}$$

where $d\Omega^2_{6-p}$ is the metric of a unit $6-p$ sphere.

The Hodge duals of the RR field strengths can be easily computed from eqs. (2.5) and (2.1). Clearly ${}^*F^{(p)}$ is a $(10-p)$-form whereas ${}^*F^{(p+2)}$ is a $(8-p)$-form. After a simple calculation one can check that ${}^*F^{(p)}$ and ${}^*F^{(p+2)}$ have the following components:

$$\begin{aligned} {}^*F^{(p)}_{x^{p-1},x^p,\rho,\phi,\theta^1\cdots,\theta^{6-p}} &= (-1)^{p+1}\,(7-p)\,\sin\varphi\, R^{7-p}\,\rho^{6-p}\, h_p\, f_p^{-1}\,\sqrt{\hat g^{(6-p)}}\ ,\\ {}^*F^{(p+2)}_{\rho,\phi,\theta^1\cdots,\theta^{6-p}} &= (-1)^{p+1}\,(7-p)\,\cos\varphi\, R^{7-p}\,\rho^{6-p}\,\sqrt{\hat g^{(6-p)}}\ , \end{aligned} \tag{2.9}$$

where $\theta^1,\cdots,\theta^{6-p}$ are coordinates of the unit $6-p$ sphere and $\hat g^{(6-p)}$ is the determinant of the S^{6-p} metric. These forms satisfy the equations:

$$d^*\,F^{(p)} \,=\, H\,\wedge\,{}^*\,F^{(p+2)}\ , \qquad\qquad d^*\,F^{(p+2)} \,=\, 0\ . \tag{2.10}$$

Then, one can represent $^*F^{(p)}$ and $^*F^{(p+2)}$ in terms of three potentials as follows:

$$\begin{aligned} {}^*F^{(p)} &= dC^{(9-p)} - H \wedge C^{(7-p)} \,, \\ {}^*F^{(p+2)} &= dC^{(7-p)} - H \wedge C^{(5-p)} \,, \end{aligned} \tag{2.11}$$

where $H = dB$. In eq. (2.11) $C^{(r)}$ is an r-form. Actually only for $p = 3$ the term $H \wedge C^{(5-p)}$ in the second of these equations gives a non-vanishing contribution. By a direct calculation one can check that the other two potentials $C^{(7-p)}$ and $C^{(9-p)}$ have the components:

$$\begin{aligned} C^{(7-p)}_{\phi,\theta^1\cdots,\theta^{6-p}} &= (-1)^{p+1} \cos\varphi \, R^{7-p} \, \rho^{7-p} \sqrt{\hat{g}^{(6-p)}} \,, \\ C^{(9-p)}_{x^{p-1},x^p,\phi,\theta^1\cdots,\theta^{6-p}} &= (-1)^{p+1} \sin\varphi \, R^{7-p} \, \rho^{7-p} \, h_p \, f_p^{-1} \sqrt{\hat{g}^{(6-p)}} \,. \end{aligned} \tag{2.12}$$

Apart from the ones displayed in eqs. (2.5) and (2.9), the RR field strengths and their duals have another components. Actually, due to the identification of $F^{(p)}$ and $^*F^{(10-p)}$, the forms appearing in these equations are not all different. Using this fact it is not difficult to obtain all the components of the RR gauge forms. From this result one can check that eq. (2.10) is satisfied and, thus, the representation of $^*F^{(p)}$ and $^*F^{(p+2)}$ in terms of the different potentials appearing on the right-hand side of eq. (2.11) holds (although these potentials have another components in addition to the ones written in eq. (2.12)). Let us specify this for $p = 3$. In this case one can easily check that $F^{(3)}$ and $F^{(5)}$ are given by:

$$\begin{aligned} F^{(3)} &= \sin\varphi \, \partial_r f_3^{-1} \, dx^0 \wedge dx^1 \wedge dr \,, \\ F^{(5)} &= \cos\varphi \Big[h_3 \partial_r f_3^{-1} dx^0 \wedge \cdots \wedge dx^3 \wedge dr \, + \, 4R^4 \rho^3 d\rho \wedge d\phi \wedge \epsilon_{(3)} \Big] \,. \end{aligned} \tag{2.13}$$

Moreover, $^*F^{(3)}$ and $^*F^{(5)} = F^{(5)}$ can be represented in terms of three RR potentials $C^{(6)}$, $C^{(4)}$ and $C^{(2)}$ as in eq. (2.11) with:

$$\begin{aligned} C^{(6)} &= \sin\varphi \, R^4 \, \rho^4 \, h_3 f_3^{-1} \, dx^2 \wedge dx^2 \wedge d\phi \wedge \epsilon_{(3)} \,, \\ C^{(4)} &= \cos\varphi \Big[h_3 f_3^{-1} dx^0 \wedge \cdots \wedge dx^3 \, + \, R^4 \rho^4 d\phi \wedge \epsilon_{(3)} \Big] \,, \\ C^{(2)} &= -\sin\varphi f_3^{-1} dx^0 \wedge dx^1 \,, \end{aligned} \tag{2.14}$$

where $\epsilon_{(3)}$ is the volume form of the unit S^3 (with this election of $C^{(2)}$ one has $F^{(3)} = -dC^{(2)}$). One can treat similarly the other cases.

2.1 The brane probe

Let us now embed a D(8-p)-brane in the near-horizon region of the (D(p-2), Dp) geometry. In this region r is small and one can approximate the harmonic function f_p appearing in the supergravity solution as:

$$f_p \approx \frac{R^{7-p}}{r^{7-p}} \,. \tag{2.15}$$

The D(8-p)-brane probe we will be dealing with wraps the (6-p) transverse sphere and extends along the $x^{p-1}x^{p}$ directions. The response of the probe to the background is determined by its action S, which is the sum of a Dirac-Born-Infeld and a Wess-Zumino term [15]:

$$S \,=\, S_{DBI} \,+\, S_{WZ} \;. \tag{2.16}$$

The Dirac-Born-Infeld term S_{DBI} is:

$$S_{DBI} \,=\, -T_{8-p} \int d^{9-p}\xi \, e^{-\tilde{\phi}_D} \sqrt{-\det \left(\, g \,+\, \mathcal{F} \, \right)} \;, \tag{2.17}$$

where g is the induced worldvolume metric, T_{8-p} is the tension of the D(8-p)-brane:

$$T_{8-p} \,=\, (2\pi)^{p-8} \, (\, \alpha^{\,\prime} \,)^{\frac{p-9}{2}} \, (\, g_s \,)^{-1} \;, \tag{2.18}$$

and, if $P[\cdots]$ denotes the pullback to the worldvolume of a bulk field, $\mathcal{F}$ is given by:

$$\mathcal{F} \,=\, F \,-\, P[B] \,=\, dA \,-\, P[B] \;, \tag{2.19}$$

with F being a $U(1)$ worldvolume gauge field strength and A its potential. The Wess-Zumino term of the action S_{WZ} couples the probe to the RR potentials of the background. For the brane probe configuration we are considering and the (D(p-2), Dp) background, S_{WZ} is given by:

$$S_{WZ} \,=\, T_{8-p} \int \Big[\; P[C^{(9-p)}] \,+\, \mathcal{F} \wedge P[C^{(7-p)}] \; \Big] \;. \tag{2.20}$$

The worldvolume coordinates ξ^{α} ($\alpha = 0, \cdots, 8-p$) will be taken as:

$$\xi^{\alpha} \,=\, (t, x^{p-1}, x^{p}, \theta^{1}, \cdots, \theta^{6-p}) \;. \tag{2.21}$$

As we will confirm soon, the set of coordinates (2.21) is quite convenient to study the kind of configurations we are interested in. These configurations are embeddings of the D(8-p)-brane which, in our system of coordinates, are described by functions of the type:

$$r \,=\, r(t) \;, \qquad \rho \,=\, \rho(t) \;, \qquad \phi \,=\, \phi(t) \;. \tag{2.22}$$

Let us now evaluate the action for the ansatz (2.22). We shall begin by studying the Wess-Zumino term. In this term only the components of $C^{(7-p)}$ and $C^{(9-p)}$ written in eq. (2.12) contribute. Actually, it is easy to see that the pullback of the RR potential $C^{(7-p)}$ is coupled to the $x^{p-1}x^{p}$ component of $\mathcal{F}$. Assuming that $\mathcal{F}_{x^{p-1},x^{p}}$ is independent of the angles $\theta^{1} \cdots \theta^{6-p}$, one gets that S_{WZ} can be written as:

$$S_{WZ} \,=\, T_{8-p} \, \Omega_{6-p} \, R^{7-p} \, \cos\varphi \int dt \, dx^{p-1} \, dx^{p} \, \rho^{7-p} \, (-1)^{p+1} \, \dot{\phi} \, \Big[\, \mathcal{F}_{x^{p-1},x^{p}} \,+\, h_p f_p^{-1} \tan\varphi \, \Big] \;, \tag{2.23}$$

where $\dot{\phi} = d\phi/dt$ and Ω_{6-p} is the volume of the S^{6-p} sphere, given by:

$$\Omega_{6-p} \,=\, \frac{2\pi^{\frac{7-p}{2}}}{\Gamma\Big(\frac{7-p}{2}\Big)} \;. \tag{2.24}$$

Notice that for the ansatz (2.22) the scalars r, ρ and ϕ do not depend on the coordinates x^{p-1} and x^p and, therefore, $P[B]$ has only non-zero components along the directions $x^{p-1}x^p$. By using the actual value of the B field for the (D(p-2), Dp) background in the definition of $\mathcal{F}$ (eq. (2.19)), one immediately gets that the term inside the square brackets on the right-hand side of eq. (2.23) is:

$$\mathcal{F}_{x^{p-1},x^p} \,+\, h_p f_p^{-1} \tan\varphi \,=\, F_{x^{p-1},x^p} \,. \tag{2.25}$$

In what follows we will assume that the only non-zero component of the worldvolume gauge field is F_{x^{p-1},x^p} and we will denote from now on $\mathcal{F}_{x^{p-1},x^p}$ and F_{x^{p-1},x^p} simply by $\mathcal{F}$ and F respectively. Then, the total action can be written as:

$$S \,=\, \int dt\, dx^{p-1}\, dx^p\, \mathcal{L} \,, \tag{2.26}$$

where the lagrangian density $\mathcal{L}$ is given by:

$$\begin{aligned}\mathcal{L} \,&=\, T_{8-p}\,\Omega_{6-p}\,R^{7-p}\,\times \\ &\times\Big[\,-\,\rho^{6-p}\lambda_1\sqrt{r^{-2}\,f_p^{-1}\,-\,r^{-2}\dot r^2\,-\,\frac{\dot\rho^2}{1-\rho^2}\,-\,(1-\rho^2)\,\dot\phi^2}\,+\,\lambda_2\,(-1)^{p+1}\rho^{7-p}\,\dot\phi\,\Big]\,.\end{aligned} \tag{2.27}$$

In eq. (2.27) we have introduced the functions λ_1 and λ_2, which are defined as:

$$\lambda_1 \,=\, \sqrt{h_p f_p^{-1} \,+\, \mathcal{F}^2\, h_p^{-1}}\,, \qquad\qquad \lambda_2 \,=\, F\cos\varphi \,. \tag{2.28}$$

Before starting the analysis of the lagrangian density (2.27), let us discuss how the brane probe is extended along the $x^{p-1}x^p$ directions. The main motivation to extend the probe along these directions is to allow the worldvolume of the brane to capture the flux of the B field of the background. It is thus natural to characterize the spreading of the D(8-p)-brane in the $x^{p-1}x^p$ plane by means of the flux of the F field. Accordingly we will extend our brane along the $x^{p-1}\,x^p$ directions in such a way that there are N' units of worldvolume flux. *i.e.*:

$$\int\, dx^{p-1}\, dx^p\, F \,=\, \frac{2\pi}{T_f}\, N' \,, \tag{2.29}$$

with $T_f \,=\, (2\pi\alpha')^{-1}$ being the fundamental string tension. Clearly, given F (which will be determined below), eq. (2.29) gives the volume occupied by the brane probe in the noncommutative plane in terms of the flux number N'.

Let us now resume our study of the dynamics of the system by performing a canonical hamiltonian analysis of the lagrangian density (2.27). First of all, for simplicity, let us absorb the sign $(-1)^{p+1}$ sign of the Wess-Zumino term of $\mathcal{L}$ by redefining $\dot\phi$ if necessary. The density of momenta associated to $\mathcal{L}$ are:

$$\mathcal{P}_r \,=\, \frac{\partial\mathcal{L}}{\partial\dot r} \,\equiv\, T_{8-p}\,\Omega_{6-p}\,R^{7-p}\,\lambda_1\,\pi_r\,,$$

$$
\begin{aligned}
\mathcal{P}_\rho &= \frac{\partial \mathcal{L}}{\partial \dot\rho} \equiv T_{8-p}\,\Omega_{6-p}\,R^{7-p}\,\lambda_1\,\pi_\rho\ ,\\
\mathcal{P}_\phi &= \frac{\partial \mathcal{L}}{\partial \dot\phi} \equiv T_{8-p}\,\Omega_{6-p}\,R^{7-p}\,\lambda_1\,\pi_\phi\ ,
\end{aligned}
\tag{2.30}
$$

where we have defined the reduced momenta π_r, π_ρ and π_ϕ. By using the explicit value of of $\mathcal{L}$, given in eq. (2.27), we get:

$$
\begin{aligned}
\pi_r &= \frac{\rho^{6-p}}{r^2}\,\frac{\dot r}{\sqrt{r^{-2}\,f_p^{-1}\, -\, r^{-2}\dot r^2\, -\, \frac{\dot\rho^2}{1-\rho^2}\, -\, (1-\rho^2)\,\dot\phi^2}}\ ,\\
\pi_\rho &= \frac{\rho^{6-p}}{1-\rho^2}\,\frac{\dot \rho}{\sqrt{r^{-2}\,f_p^{-1}\, -\, r^{-2}\dot r^2\, -\, \frac{\dot\rho^2}{1-\rho^2}\, -\, (1-\rho^2)\,\dot\phi^2}}\ ,\\
\pi_\phi &= (1-\rho^2)\rho^{6-p}\,\frac{\dot \phi}{\sqrt{r^{-2}\,f_p^{-1}\, -\, r^{-2}\dot r^2\, -\, \frac{\dot\rho^2}{1-\rho^2}\, -\, (1-\rho^2)\,\dot\phi^2}}\, +\, \Lambda\,\rho^{7-p}\ ,
\end{aligned}
\tag{2.31}
$$

where, in the third of these expressions we have introduced the quantity Λ, defined as:

$$
\Lambda = \frac{\lambda_2}{\lambda_1}\ . \tag{2.32}
$$

The hamiltonian density of the system can be obtained in the standard way, namely:

$$
\mathcal{H} = \dot r\,\mathcal{P}_r + \dot\rho\,\mathcal{P}_\rho + \dot\phi\mathcal{P}_\phi - \mathcal{L} \equiv T_{8-p}\,\Omega_{6-p}\,R^{7-p}\,\lambda_1\,h\ , \tag{2.33}
$$

where, in analogy with what we have done for the momenta, we have defined the reduced quantity h. From eqs. (2.27) and (2.31), one gets after a simple calculation that h is given by:

$$
h = r^{-1}\,f_p^{-\frac{1}{2}}\left[r^2\,\pi_r^2 + \rho^{2(6-p)} + (1-\rho^2)\,\pi_\rho^2 + \frac{\left(\pi_\phi - \Lambda\rho^{7-p}\right)^2}{1-\rho^2}\right]^{\frac{1}{2}}\ . \tag{2.34}
$$

2.2 Fixed size configurations

We would like to obtain solutions of the equations of motion derived from the hamiltonian (2.34) which correspond to a brane of fixed size. It follows from eq. (2.8) that the coordinate ρ plays the role of the size of the system on the S^{6-p} sphere. For this reason it is interesting to look at solutions of the equations of motion for which ρ is constant. This same problem was analyzed in ref. [7] for the case of brane probes moving in the near-horizon Dp-brane background (see also refs. [3, 4, 5, 8]). In ref. [7] it was found that the ρ-dependent terms in the hamiltonian can be arranged to produce a sum of squares from which the $\rho =$ constant solutions can be found by inspection. By comparing the right-hand side of eq. (2.34) with the corresponding expression in ref. [7], one immediately realizes that the same kind of arrangement can be done in our case if the condition:

$$\Lambda \,=\, 1 \,, \tag{2.35}$$

is satisfied. Indeed, if eq. (2.35) holds, one can rewrite h as:

$$h \,=\, r^{-1}\, f_p^{-\frac{1}{2}} \left[\pi_\phi^2 \,+\, r^2\, \pi_r^2 \,+\, (1-\rho^2)\,\pi_\rho^2 \,+\, \frac{\left(\pi_\phi \rho - \rho^{6-p}\right)^2}{1-\rho^2} \right]^{\frac{1}{2}} \,. \tag{2.36}$$

The analysis of the hamiltonian (2.36) can be performed along the same lines as in ref. [7] (see below). Before carrying out this study, let us explore the content of the condition (2.35). By recalling the definition of Λ (eq. (2.32)), we realize that eq. (2.35) is equivalent to require that $\lambda_1 = \lambda_2$. Moreover, by using eq. (2.25) in the definition of λ_1 (eq. (2.28)), one obtains:

$$\lambda_1^2 \,=\, \cos^2\varphi\, F^2 \,+\, f_p^{-1} \left(F\sin\varphi \,-\, \frac{1}{\cos\varphi} \right)^2 \,. \tag{2.37}$$

By comparing the right-hand side of eq. (2.37) with the definition of λ_2 (eq. (2.28)), one concludes immediately that the condition $\Lambda = 1$ is equivalent to have the following constant value of the worldvolume gauge field F:

$$F \,=\, \frac{1}{\sin\varphi\cos\varphi} \,=\, 2\csc(2\varphi) \,. \tag{2.38}$$

By substituting this value of F on the right-hand side of eq. (2.37) one gets that λ_1 is also constant and given by:

$$\lambda_1 \,=\, \frac{1}{\sin\varphi} \,. \tag{2.39}$$

In the next section we will show that for the value of F displayed in eq. (2.38) the brane probe breaks the supersymmetry of the background exactly in the same way as a massless particle. In this section this value of the worldvolume gauge field should be considered as an ansatz which allows us to find a class of fixed size solutions of the equations of motion of the brane which are particularly interesting. In the remaining of this section we will assume that F is given by (2.38).

It is now straightforward to find the configurations of the system with constant ρ. From the second expression in eq. (2.31) one concludes that one must have:

$$\pi_\rho \,=\, 0 \,. \tag{2.40}$$

Then, the hamiltonian equation of motion for π_ρ, *i.e.* $\dot\pi_\rho \,=\, -\,\partial h/\partial\rho$, implies that the last term on the right-hand side of eq. (2.36) must vanish. For $p<6$ this happens either for:

$$\rho = 0 \,, \tag{2.41}$$

or else when π_ϕ is given by:

$$\pi_\phi \,=\, \rho^{5-p} \,. \tag{2.42}$$

(when $p = 6$ only eq. (2.42) gives rise to a constant ρ configuration). Notice that, as h does not depend on ϕ, the momentum π_ϕ is a constant of motion and, thus, for $p \neq 5$, eq. (2.42) only makes sense when ρ is constant. Moreover, when $p \neq 5$, eq. (2.42) determines ρ in terms of π_ϕ. On the contrary, when $p = 5$ eq. (2.42) fixes $\pi_\phi = 1$, independently of the value of ρ.

It is not difficult to translate the constant ρ condition into a relation involving the time derivatives of ϕ and r. Let us, first of all, invert the relation (2.31) between π_ϕ and $\dot\phi$. By taking $\Lambda = 1$ as in eq. (2.35) one easily finds after a simple calculation that:

$$\dot\phi = \frac{\pi_\phi - \rho^{7-p}}{1-\rho^2} \frac{\left[r^{-2}(f_p^{-1} - \dot r^2) - \frac{\dot\rho^2}{1-\rho^2} \right]^{\frac{1}{2}}}{\left[\pi_\phi^2 + \frac{\left(\pi_\phi \rho - \rho^{6-p}\right)^2}{1-\rho^2} \right]^{\frac{1}{2}}} \, . \tag{2.43}$$

By taking $\dot\rho = 0$ on eq. (2.43), and imposing one of the two conditions (2.41) or (2.42), one gets that in both cases $\dot\phi$ and $\dot r$ satisfy the following relation:

$$f_p \, [\, r^2 \, \dot\phi^2 + \dot r^2 \,] = 1 \, . \tag{2.44}$$

For the configurations we are considering the last two terms of the reduced hamiltonian h of eq. (2.36) vanish and, then, our configurations certainly minimize the energy. These configurations are characterized by eq. (2.38), which fixes the value of the worldvolume gauge field, and (2.44), which is a consequence of the vanishing of the last two terms of the hamiltonian. Remarkably, eq. (2.44) is the condition satisfied by a particle moving in the (r, ϕ) plane at $\rho = 0$ along a null trajectory, *i.e.* with $ds^2 = 0$, in the metric (2.1). Thus, our brane probe configurations have the characteristics of a massless particle : the so-called giant graviton. Notice that the point $\rho = 0$ can be considered as the "center of mass" of the expanded brane. In order to confirm this picture let us introduce the two-component vector $\mathbf{v}$, defined as:

$$\mathbf{v} = (v^{\underline{r}}, v^{\underline{\phi}}) \equiv f_p^{\frac{1}{2}} \, (\dot r \, , \, r\dot\phi) \, , \tag{2.45}$$

which is nothing but the velocity of the particle in the (r, ϕ) plane. From eq. (2.44) one clearly has:

$$(v^{\underline{r}})^2 + (v^{\underline{\phi}})^2 = 1 \, , \tag{2.46}$$

which simply states that the center of mass of the giant graviton moves at the speed of light. The corresponding value of the momentum density $\mathcal{P}_\phi$ can be straightforwardly obtained from eqs. (2.42) and (2.30). Indeed, by using eqs. (2.18), (2.24), (2.3) and (2.39), one gets:

$$\mathcal{P}_\phi = \frac{T_f}{2\pi} \, F \, N \, \rho^{5-p} \, , \tag{2.47}$$

where F is given in eq. (2.38). The momenta p_ϕ and p_r can be obtained from the densities $\mathcal{P}_\phi$ and $\mathcal{P}_r$ by integrating them in the $x^{p-1}x^p$ plane:

$$p_\phi = \int dx^{p-1} \, dx^p \; \mathcal{P}_\phi \, , \qquad\qquad p_r = \int dx^{p-1} \, dx^p \; \mathcal{P}_r \, . \tag{2.48}$$

By using the value of the momentum density $\mathcal{P}_\phi$ given in eq. (2.47) and the flux condition (2.29), one gets the following value of p_ϕ for a giant graviton:

$$p_\phi \,=\, N\,N'\,\rho^{5-p} \ . \tag{2.49}$$

It follows from eq. (2.49) that, when $p < 5$, the size ρ of the wrapped brane increases with the momentum p_ϕ. Moreover, it is interesting to point out that, as $0 \le \rho \le 1$, when $p < 5$ the momentum p_ϕ has a maximum value p_ϕ^{max} given by:

$$p_\phi^{max} \,=\, N\,N' \ , \tag{2.50}$$

which is reached when $\rho = 1$. The existence of such a maximum for p_ϕ is a manifestation of the so-called stringy exclusion principle. Notice that when $p = 5$ the momentum p_ϕ is independent of ρ, whereas for $p = 6$ the value of p_ϕ written in eq. (2.50) is in fact a minimum.

Let us now study the energy of the giant graviton solution. First of all, we define the metric elements $\mathcal{G}_{MN}$ as:

$$\mathcal{G}_{MN} \,=\, G_{MN}\big|_{\rho=0} \ , \tag{2.51}$$

where the G_{MN}'s correspond to the metric displayed in eqs. (2.1) and (2.8). The hamiltonian for the giant graviton configurations, which we will denote by H_{GG}, can be easily obtained from eq. (2.36). In terms of the $\rho = 0$ metric it can be written as:

$$H_{GG} \,=\, \sqrt{-\mathcal{G}_{tt}}\,\Bigg[\,\frac{p_\phi^2}{\mathcal{G}_{\phi\phi}} \,+\, \frac{p_r^2}{\mathcal{G}_{rr}}\,\Bigg]^{\frac{1}{2}} \ , \tag{2.52}$$

which, according to our expectations, is exactly the hamiltonian of a massless particle which moves in the metric $\mathcal{G}_{MN}$ along a trajectory contained in the (r, ϕ) plane. Interestingly, one can use the hamiltonian (2.52) to find and solve the equations of motion for the giant graviton. Let us, first of all, rewrite H_{GG} as:

$$H_{GG} \,=\, R^{\frac{p-7}{2}}\,\Big[\,r^{7-p}\,p_r^2 \,+\, r^{5-p}\,p_\phi^2\,\Big]^{\frac{1}{2}} \ . \tag{2.53}$$

We will study the equations of motion of this system by using the conservation of energy. In this method we first put $H_{GG} \,=\, E$, for constant E, and then we use the relation between p_r and $\dot r$, namely:

$$p_r \,=\, \frac{R^{7-p}}{r^{7-p}}\,E\,\dot r \ . \tag{2.54}$$

By substituting (2.54) in the condition $H_{GG} \,=\, E$, we get:

$$\dot r^2 \,+\, \frac{r^{7-p}}{R^{7-p}}\,\Bigg[\,\frac{p_\phi^2}{E^2\,R^{7-p}}\,r^{5-p} \,-\, 1\,\Bigg] \,=\, 0 \ . \tag{2.55}$$

Eq. (2.55) determines the range of values that r can take. Indeed, by consistency of eq. (2.55), the second term in this equation must be negative or null. The points for which this term vanishes are the turning points of the system. For $p<5$ these points are $r=0$ and r_*, with r_* being:

$$(r_*)^{5-p} \,=\, \frac{E^2}{p_\phi^2}\, R^{7-p} \,. \tag{2.56}$$

In this $p<5$ case, r can take values in the range $0\le r\le r_*$. If $p=5$ only the $r=0$ turning point exists and r is unrestricted, *i.e.* r can take any non-negative value. Finally if $p=6$ the $r=0$ turning point is missing and $r\ge r_*$, where r_* is the value given in eq. (2.56) for $p=6$.

It is not difficult to find the explicit dependence of r on t. Let us consider first the $p\neq 5$ case. From eq. (2.55) it follows that t as a function of r is given by the following indefinite integral:

$$t\,-\,t_* \,=\, R^{\frac{7-p}{2}} \int \frac{dr}{r^{\frac{7-p}{2}}\sqrt{1\,-\,\big(\frac{r}{r_*}\big)^{5-p}}}\;, \qquad (\,p\neq 5\,)\,, \tag{2.57}$$

where t_* is a constant of integration. The integral (2.57) can be easily performed by means of the following trigonometric change of variables:

$$\Big(\frac{r}{r_*}\Big)^{5-p} \,=\, \cos^2\theta\,, \qquad (\,p\neq 5\,)\,. \tag{2.58}$$

The result of the integration is:

$$\Big(\frac{r_*}{r}\Big)^{5-p} \,=\, 1\,+\,(5-p)^2\,(r_*)^{5-p}\,R^{p-7}\,\Big(\frac{t\,-\,t_*}{2}\Big)^2\,, \qquad (\,p\neq 5\,)\,. \tag{2.59}$$

It follows from eq. (2.59) that t_* is precisely the value of t at which $r=r_*$. Moreover if $p<5$ the coordinate $r\to 0$ as $t\,-\,t_*\to\pm\infty$, *i.e.* the giant graviton always falls asymptotically into the black hole. On the contrary, for $p=6$ the coordinate r diverges asymptotically and, thus, in this case the particle always escapes away from the $r=0$ point.

The $p=5$ case needs a special treatment since, in this case, eq. (2.57) is not valid any more. One can, however, easily integrate eq. (2.55) with the result:

$$r\,=\,r_0\,e^{\pm\frac{t}{R}\sqrt{1-\frac{p_\phi^2}{E^2R^2}}}\,, \qquad (\,p=5\,)\,. \tag{2.60}$$

It follows from (2.60) that, in this case, the solution connects asymptotically the points $r=0$ and $r=\infty$.

In order to complete the integration of the equations of motion one has to determine ϕ as a function of t. This can be easily achieved by substituting $r(t)$ from eq. (2.59) or (2.60) into the condition (2.44) to get $\dot\phi$, followed by an integration over t. The result for $p\neq 5$ is:

$$\tan\Big[\,\frac{5-p}{2}\,(\phi\,-\,\phi_*)\Big] \,=\, \frac{5-p}{2}\,\Big(\frac{r_*}{R}\Big)^{\frac{5-p}{2}}\;\frac{t-t_*}{R}\,, \qquad (\,p\neq 5\,)\,, \tag{2.61}$$

whereas for $p=5$ one gets:

$$\phi\,=\,\phi_0\,+\,\frac{p_\phi}{E\,R^2}\,t\,, \qquad (\,p=5\,)\,. \tag{2.62}$$

It is also interesting to find the value of the velocity $\mathbf{v}$ along the trajectory. Actually, a simple calculation shows that, for $p \neq 5$, it depends on the coordinate r as:

$$v^{\underline{r}} = -\Big[1 - \Big(\frac{r}{r_*}\Big)^{5-p}\Big]^{\frac{1}{2}} , \qquad v^{\underline{\phi}} = \Big(\frac{r}{r_*}\Big)^{\frac{5-p}{2}} , \qquad (p \neq 5) , \tag{2.63}$$

where, on the right-hand side, it should be understood that r is the function of t given in eq. (2.59) for $t \geq t_*$. Curiously, when $p = 5$ the vector $\mathbf{v}$ is constant and has the following components:

$$v^{\underline{r}} = \pm\Big[1 - \frac{p_\phi^2}{E^2 R^2}\Big]^{\frac{1}{2}} , \qquad v^{\underline{\phi}} = \frac{p_\phi}{E\,R} , \qquad (p = 5) . \tag{2.64}$$

To finish this section, let us now discuss the extensivity of the brane probe on the noncommutative plane $x^{p-1}\,x^p$. As argued above, for fixed worldvolume flux N', this extensivity depends on the value of the gauge field F. Actually, by using in eq. (2.29) the value of F given in eq. (2.38), one gets that the volume occupied by the brane probe along the $x^{p-1}\,x^p$ plane is:

$$\int dx^{p-1}\,dx^p = \frac{\pi N'}{T_f}\,\sin(2\varphi) . \tag{2.65}$$

This volume clearly goes to zero as $\varphi \to 0$ for fixed N'. This means that if we switch off the B field in the background, the D(8-p)-brane probe is effectively converted into a D(6-p)-brane, in agreement with the results of refs. [3]-[7]. Actually, one can check that in this limit our results agree with those corresponding to the Dp-brane background if we replace everywhere N by NN'.

3 Supersymmetry

The objective of this section is to analyze the supersymmetry behaviour of the brane configurations studied in section 2. Actually, we will verify that these configurations break the supersymmetry of the background just as a massless particle which moves precisely along the trajectories found in section 2.

The number of supersymmetries preserved by a Dp-brane is the number of independent solutions of the equation [16]:

$$\Gamma_\kappa\,\epsilon = \epsilon , \tag{3.1}$$

where ϵ is a Killing spinor of the background and Γ_κ is the so-called κ-symmetry matrix, which depends on the background and on the type of brane. For a Dp-brane in a type IIB background Γ_κ is given by [16]:

$$\begin{aligned}\Gamma_\kappa &= \frac{1}{\sqrt{-\det(g + \mathcal{F})}} \sum_{n=0}^{\infty} \frac{1}{2^n\,n!}\,\gamma^{\mu_1\nu_1\cdots\mu_n\nu_n} \times \\ &\quad \times \mathcal{F}_{\mu_1\nu_1}\cdots\mathcal{F}_{\mu_n\nu_n}\,J^{(n)} ,\end{aligned} \tag{3.2}$$

where g is the induced metric on the brane worldvolume, $\mathcal{F} = F - P[B]$ and $J^{(n)}$ is the following matrix:

$$J^{(n)} \,=\, (-1)^n \,\sigma_3^{\frac{p-3}{2}+n}\,(i\sigma_2)\,\otimes\,\Sigma_0\;, \tag{3.3}$$

with Σ_0 being:

$$\Sigma_0 \,=\, \frac{1}{(p+1)!}\;\epsilon^{\mu_1\cdots\mu_{p+1}}\,\gamma_{\mu_1\cdots\mu_{p+1}}\;. \tag{3.4}$$

Recall that in the type IIB theory the spinor ϵ is actually composed by two Majorana-Weyl spinors ϵ_1 and ϵ_2 of well defined ten-dimensional chirality, which can be arranged as a two-component vector in the form:

$$\epsilon \,=\, \begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix}\;. \tag{3.5}$$

The Pauli matrices appearing in the expression of $J^{(n)}$ act on this two-dimensional vector. Moreover, in eqs. (3.2) and (3.3) $\gamma_{\mu_1\mu_2\cdots}$ are antisymmetrized products of the induced world-volume Dirac matrices which, in terms of the ten-dimensional constant gamma matrices $\Gamma_{\underline{M}}$, are given by:

$$\gamma_\mu \,=\, \partial_\mu\,X^M\,E_M^{\underline{M}}\,\Gamma_{\underline{M}}\;, \tag{3.6}$$

where $E_M^{\underline{M}}$ is the ten-dimensional vielbein. There exist similar expressions for the type IIA theory. Actually, for the sake of simplicity, we will restrict ourselves to the analysis of the $(D1, D3)$ background, although one can find similar results for the general case by making the appropriate modifications in our equations.

3.1 Killing spinors of the $(\mathbf{D1}, \mathbf{D3})$ background

In order to solve eq. (3.1) it is clear that we must first determine the Killing spinors of the background, which is equivalent to characterize the supersymmetry preserved by our supergravity solution. We are considering bosonic backgrounds, which are supersymmetric iff the supersymmetry variation of the fermionic supergravity fields vanishes. Actually, this only occurs for some class of transformation parameters, which are precisely the Killing spinors we are interested in.

The analysis of the supersymmetry preserved by the (D1,D3) background by using the transformation rules of the type IIB supergravity is performed in appendix A. Here we will characterize the Killing spinors of this background by means of an alternative and more simplified method [17], which makes use of the κ-symmetry matrix (3.2). Let us place a test D3-brane parallel to the background and choose its worldvolume coordinates as $(\xi^0, \xi^i) \,=\, (t, x^i)$ for $i \,=\, 1, 2, 3$. The equations which determine the embedding of the parallel D3-brane are :

$$X^0 \,=\, t \,=\xi^0\,, \qquad\qquad X^i \,=\, x^i \,=\xi^i\;, \qquad (i \,=1,2,3)\;, \tag{3.7}$$

with the other spacetime coordinates being independent of the (ξ^0, ξ^i). We will study the supersymmetry preserved by this test brane when the worldvolume gauge field F is zero, *i.e.* when $\mathcal{F} \,=\, -P[B]$. This means that the only non-zero component of $\mathcal{F}$ is:

$$\mathcal{F}_{x^2x^3} \,=\, -\tan\varphi\, f_3^{-1}\, h_3\;. \tag{3.8}$$

The test brane configuration we are considering is the same as the one that creates the background. Therefore, it is natural to expect that such a parallel test brane preserves the same supersymmetries as the supergravity background. Then, let us consider the κ-symmetry condition $\Gamma_k \epsilon = \epsilon$ for this case. The induced gamma matrices for the embedding (3.7) are:

$$\begin{aligned}\gamma_{x^\alpha} &= f_3^{-\frac{1}{4}}\,\Gamma_{\underline{x^\alpha}}\,, \qquad\qquad (\alpha = 0,1)\,,\\ \gamma_{x^i} &= f_3^{-\frac{1}{4}}\,h_3^{\frac{1}{2}}\,\Gamma_{\underline{x^i}}\,, \qquad\qquad (i = 2,3)\,.\end{aligned} \tag{3.9}$$

By using eq. (3.9) in the expression of Γ_k for $p = 3$ (eqs. (3.2)-(3.4)), one gets that the κ-symmetry matrix for this case is:

$$\Gamma_k = h_3^{\frac{1}{2}}\Big[\cos\varphi\,(i\sigma_2)\,\Gamma_{\underline{x^0\cdots x^3}} - f_3^{-\frac{1}{2}}\sin\varphi\,\sigma_1\Gamma_{\underline{x^0x^1}}\Big]\,. \tag{3.10}$$

In eq. (3.10), and in what follows, we have suppressed the tensor product symbol. It is now straightforward to check that the condition $\Gamma_k\epsilon = \epsilon$ can be written as:

$$(i\sigma_2)\,\Gamma_{\underline{x^0\cdots x^3}}\,\epsilon = e^{-\alpha\,\Gamma_{\underline{x^2x^3}}\sigma_3}\,\epsilon\,, \tag{3.11}$$

where α is the function of r given by:

$$\sin\alpha = f_3^{-\frac{1}{2}}\,h_3^{\frac{1}{2}}\sin\varphi\,, \qquad\qquad \cos\alpha = h_3^{\frac{1}{2}}\cos\varphi\,. \tag{3.12}$$

In appendix A we will verify that the supersymmetry invariance of the dilatino of type IIB supergravity in the $(D1, D3)$ background requires that the supersymmetry parameter ϵ satisfies eq. (3.11) with α given by eq. (3.12), which is is a confirmation of the correctness of our κ-symmetry argument. Moreover, in view of eq. (3.11), the Killing spinor ϵ can be written as:

$$\epsilon = e^{\frac{\alpha}{2}\Gamma_{\underline{x^2x^3}}\sigma_3}\,\tilde{\epsilon}\,, \tag{3.13}$$

where $\tilde{\epsilon}$ is a spinor which satisfies:

$$(i\sigma_2)\,\Gamma_{\underline{x^0\cdots x^3}}\,\tilde{\epsilon} = \tilde{\epsilon}\,. \tag{3.14}$$

In our study of the supersymmetry preserved by the giant graviton solution we will need to know the dependence of ϵ on the coordinate ρ. Again, this dependence can be extracted by solving the corresponding supergravity equations. Here, however, we will present a simpler argument which, as we will check in appendix A, gives the right answer. The starting point of our argument is to consider the supergravity solution in the asymptotic region $r \to \infty$ without making the near-horizon approximation (2.15). In this case the harmonic function f_p is given by eq. (2.2) and, when $r \to \infty$, the metric is flat and the different forms vanish. The supersymmetry preserved in this region is just the one corresponding to covariantly constant spinors, *i.e.* spinors which satisfy $D_M\epsilon = 0$ in the flat asymptotic metric. Taking $M = \rho$, we get that:

$$\partial_\rho\epsilon = -\frac{1}{4}\,\omega_\rho^{\underline{MN}}\,\Gamma_{\underline{MN}}\,\epsilon\,, \tag{3.15}$$

where $\omega_{\rho}^{\underline{MN}}$ are the components of the spin connection. As the only non-zero component of the form $\omega_{\rho}^{\underline{MN}}$ of the spin connection in our coordinates (2.1) and (2.8) for the the asymptotic metric is:

$$\omega_{\rho}^{\underline{\rho r}} = \frac{1}{\sqrt{1-\rho^2}} \ , \tag{3.16}$$

we obtain from eq. (3.15) that the ρ dependence of $\tilde{\epsilon}$ can be parametrized as:

$$\tilde{\epsilon} = e^{-\frac{\beta}{2}\,\Gamma_{\underline{\rho r}}}\ \hat{\epsilon} \ , \tag{3.17}$$

with β being the following function of ρ:

$$\sin\beta = \rho \ , \qquad\qquad \cos\beta = \sqrt{1-\rho^2} \ , \tag{3.18}$$

and $\hat{\epsilon}$ satisfying the same equation as $\tilde{\epsilon}$, namely:

$$(i\sigma_2)\,\Gamma_{\underline{x^0\cdots x^3}}\ \hat{\epsilon} = \hat{\epsilon} \ . \tag{3.19}$$

The supergravity analysis of the $(D1, D3)$ background provides the explicit dependence of $\hat{\epsilon}$ on the other coordinates (see eq. (A.12)). However, in our analysis of the κ-symmetry preserved by the giant graviton solution we will only need to know that $\hat{\epsilon}$ satisfies eq. (3.19). Actually, we will rewrite this equation in a form more convenient for our purposes. Let us, first of all, recall that all spinors (and in particular $\hat{\epsilon}$) have fixed chirality and thus $\hat{\epsilon}$ satisfies:

$$\Gamma_{\underline{x^0\cdots x^3}}\,\Gamma_{\underline{\rho r}}\,\Gamma_{\underline{\phi}}\,\Gamma_*\,\hat{\epsilon} = \hat{\epsilon} \ , \tag{3.20}$$

where we have denoted:

$$\Gamma_* = \Gamma_{\underline{\theta^1\ldots\theta^3}} \ . \tag{3.21}$$

Taking eq. (3.20) into account, eq. (3.19) can be written as:

$$\Gamma_{\underline{\rho r}}\,\hat{\epsilon} = \Upsilon\,\hat{\epsilon} \ , \tag{3.22}$$

where Υ is the following matrix:

$$\Upsilon = (i\sigma_2)\ \Gamma_{\underline{\phi}}\,\Gamma_* \ . \tag{3.23}$$

Using eq. (3.22) in eq. (3.17), we can reexpress $\tilde{\epsilon}$ as:

$$\tilde{\epsilon} = e^{-\frac{\beta}{2}\,\Upsilon}\ \ \hat{\epsilon} \ , \tag{3.24}$$

which is the parametrization of $\tilde{\epsilon}$ which we will use in our analysis of the κ-symmetry for the giant graviton.

3.2 κ-symmetry of the probe

We are now ready to determine the supersymmetry preserved by the giant graviton in the $(D1, D3)$ background[1]. According to the formalism developed in section 2, we have to consider a D5-brane probe extended along the directions $(x^2, x^3, \theta^1, \theta^2, \theta^3)$. The κ-symmetry matrix Γ_κ for such a probe can be obtained from eq. (3.2) by taking $p = 5$. For a D5-brane embedding of the type (2.22) with $\dot\rho = 0$ the induced Dirac matrices are:

$$
\begin{aligned}
\gamma_{x^0} &= \sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}} + \dot\phi\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}} + \dot r\sqrt{G_{rr}}\,\Gamma_{\underline{r}} \;, \\
\gamma_{x^2x^3} &= f_3^{-\frac{1}{2}}\,h_3\,\Gamma_{\underline{x^2x^3}} \;, \\
\gamma_{\theta^i} &= f_3^{\frac{1}{4}}\,r\,\rho\,e_i^{\underline{i}}\,\Gamma_{\underline{\theta^i}} \;,
\end{aligned}
\tag{3.25}
$$

where $e_j^{\underline{i}}$ denotes the S^3 vielbein. We will assume, as in section 2, that the brane probe has a worldvolume gauge field whose only non-vanishing component is $\mathcal{F}_{x^2x^3}$, which we simply will denote by $\mathcal{F}$. By substituting the values given in eq. (3.25) in eq. (3.2) one readily verifies that the contribution of the gauge field $\mathcal{F}$ exponentiates and, as a consequence, the matrix Γ_κ can be written as:

$$
\begin{aligned}
\Gamma_\kappa = &\frac{i\sigma_2}{\sqrt{-G_{tt} - G_{\phi\phi}\dot\phi^2 - G_{rr}\dot r^2}} \times \\
&\times \Big[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}} + \dot\phi\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}} + \dot r\sqrt{G_{rr}}\,\Gamma_{\underline{r}}\Big]\,\Gamma_*\,e^{-\eta\,\Gamma_{\underline{x^2x^3}}\sigma_3} \;.
\end{aligned}
\tag{3.26}
$$

In eq. (3.26) G_{MN} denote the elements of the metric (2.1) for $p = 3$, Γ_* is the same as in eq. (3.21) and η is defined as:

$$
\sin\eta = \frac{f_3^{-\frac{1}{2}}\,h_3^{\frac{1}{2}}}{\lambda_1} \;, \qquad \cos\eta = \frac{\mathcal{F}\,h_3^{-\frac{1}{2}}}{\lambda_1} \;,
\tag{3.27}
$$

where λ_1 has been defined in eq. (2.28).

Let us now compute the action of Γ_κ on spinor ϵ, which we will parametrize as the Killing spinors of the $(D1, D3)$ background, namely (see eqs. (3.13) and (3.24)):

$$
\epsilon = e^{\frac{\alpha}{2}\,\Gamma_{\underline{x^2x^3}}\,\sigma_3}\,\tilde\epsilon = e^{\frac{\alpha}{2}\,\Gamma_{\underline{x^2x^3}}\,\sigma_3}\,e^{-\frac{\beta}{2}\,\Upsilon}\;\hat\epsilon \;,
\tag{3.28}
$$

where α and β are given in eqs. (3.12) and (3.18), Υ is the matrix written in (3.23) and $\hat\epsilon$ is independent of ρ. By using this representation, one immediately gets:

$$
\begin{aligned}
\Gamma_\kappa\,\epsilon = &\,e^{(\eta-\frac{\alpha}{2})\,\Gamma_{\underline{x^2x^3}}\,\sigma_3}\,\frac{i\sigma_2}{\sqrt{-G_{tt} - G_{\phi\phi}\dot\phi^2 - G_{rr}\dot r^2}} \times \\
&\times \Big[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}} + \dot\phi\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}} + \dot r\sqrt{G_{rr}}\,\Gamma_{\underline{r}}\Big]\,\Gamma_*\,\tilde\epsilon \;.
\end{aligned}
\tag{3.29}
$$

[1]Similar methods have been applied in refs. [18, 19] to study the supersymmetry of the baryon vertex.

Then, making use again of eq. (3.28), one concludes that the equation $\Gamma_\kappa\,\epsilon=\epsilon$ is equivalent to the following condition for $\tilde\epsilon$:

$$\frac{i\sigma_2}{\sqrt{-G_{tt}\,-\,G_{\phi\phi}\dot\phi^2\,-\,G_{rr}\dot r^2}}\Big[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0}}+\dot\phi\sqrt{G_{\phi\phi}}\,\Gamma_{\underline{\phi}}+\dot r\sqrt{G_{rr}}\,\Gamma_{\underline{r}}\Big]\,\Gamma_*\,\tilde\epsilon\,=\,e^{(\alpha-\eta)\,\Gamma_{\underline{x^2x^3}}\,\sigma_3}\,\tilde\epsilon\;. \tag{3.30}$$

Notice that in eq. (3.30) the matrix $\Gamma_{\underline{x^2x^3}}$ only appears on the right-hand side. Actually, if $\alpha=\eta$ this dependence on $\Gamma_{\underline{x^2x^3}}$ disappears. Moreover, by comparing the definitions of α and η (eqs. (3.12) and (3.27), respectively) one immediately realizes that $\alpha=\eta$ if and only if $\lambda_1\;=\;1/\sin\varphi$. On the other hand, eq. (2.37) tells us that this only happens when $F\;=\;2\csc(2\varphi)$, *i.e.* precisely when the worldvolume gauge field strength takes the same value as the one we have found in our hamiltonian analysis of section 2 (see eq. (2.38)). Let us assume that this is the case and let us try to find out what are the consequences of this fact. Actually, we will also assume that our second condition (2.44) holds. It is a simple exercise to verify that, when (2.44) is satisfied, the denominator of the left-hand side of eq. (3.30) takes the value:

$$\sqrt{-G_{tt}\,-\,G_{\phi\phi}\dot\phi^2\,-\,G_{rr}\dot r^2}\;=\;\rho\,r\,f_3^{\frac14}\,\dot\phi\;. \tag{3.31}$$

Then, for a configuration satisfying eq. (2.38) and (2.44), the equation $\Gamma_\kappa\,\epsilon=\epsilon$ becomes:

$$\Big[\sqrt{-G_{tt}}\,\Gamma_{\underline{x^0\phi}}-\dot\phi\sqrt{G_{\phi\phi}}+\dot r\sqrt{G_{rr}}\,\Gamma_{\underline{r\phi}}\Big]\,\tilde\epsilon\;=\;\rho\,r\,f_3^{\frac14}\,\dot\phi\,\Upsilon\,\tilde\epsilon\;, \tag{3.32}$$

where Υ has been defined in eq. (3.23). Let us now rewrite eq. (3.32) in terms of the $\rho=0$ spinor $\hat\epsilon$. By substituting eq. (3.24) on both sides of eq. (3.32), and using the fact that Υ anticommutes with $\Gamma_{\underline{x^0\phi}}$ and $\Gamma_{\underline{r\phi}}$, one gets:

$$\Big[f_3^{-\frac14}\,e^{\beta\,\Upsilon}\,\Gamma_{\underline{x^0\phi}}-\dot\phi\,r\,f_3^{\frac14}\,\sqrt{1-\rho^2}+\dot r\,f_3^{\frac14}\,e^{\beta\,\Upsilon}\,\Gamma_{\underline{r\phi}}\Big]\,\hat\epsilon\;=\;\rho\,r\,f_3^{\frac14}\,\dot\phi\,\Upsilon\,\hat\epsilon\;. \tag{3.33}$$

Let us consider now eq. (3.33) for the particular case $\rho=0$. When $\rho=0$ the right-hand side of eq. (3.33) vanishes and, as $\beta=0$ is also zero in this case (see eq. (3.18)), one has:

$$\Big[f_3^{-\frac14}\,\Gamma_{\underline{x^0\phi}}-\dot\phi\,r\,f_3^{\frac14}+\dot r\,f_3^{\frac14}\Gamma_{\underline{r\phi}}\Big]\,\hat\epsilon\;=\;0\;. \tag{3.34}$$

For a general value of ρ the κ-symmetry condition can be obtained by substituting in (3.33) $e^{\beta\,\Upsilon}$ by:

$$e^{\beta\,\Upsilon}\;=\;\sqrt{1-\rho^2}\;+\;\rho\,\Upsilon\;. \tag{3.35}$$

By so doing one obtains two types of terms, with and without Υ, which, amazingly, satisfy the equation independently for all ρ as a consequence of the $\rho=0$ condition (3.34). Thus, eq. (3.34) is equivalent to the κ-symmetry condition $\Gamma_\kappa\epsilon\;=\;\epsilon$ and is the constraint we have to impose to the Killing spinors of the background in order to define a supersymmetry transformation preserved by our brane probe configurations. In order to obtain a neat interpretation of (3.34), let us define the matrix Γ_v as:

$$\Gamma_v \equiv v^{\underline{r}}\Gamma_{\underline{r}} + v^{\underline{\phi}}\Gamma_{\underline{\phi}} \,, \tag{3.36}$$

where $v^{\underline{r}}$ and $v^{\underline{\phi}}$ are the components of the velocity vector $\mathbf{v}$ defined in eq. (2.45). By using eq. (2.46), which is a consequence of (2.44), one readily proves that the matrix Γ_v satisfies:

$$(\,\Gamma_v\,)^2 = 1 \,. \tag{3.37}$$

Moreover, from the explicit expression of the components of $\mathbf{v}$ (see eq. (2.45)) it is straightforward to demonstrate that the κ-symmetry condition (3.34) can be written as:

$$\Big[\Gamma_{\underline{x^0}} + \Gamma_v\Big]\,\hat{\epsilon} = 0 \,. \tag{3.38}$$

Taking into account eq. (3.37), one can rewrite eq. (3.38) in the form:

$$\Gamma_{\underline{x^0}}\,\Gamma_v\,\hat{\epsilon} = \hat{\epsilon} \,. \tag{3.39}$$

Moreover, recalling the relation (3.28) between $\hat{\epsilon}$ and ϵ, and taking into account that $\Gamma_{\underline{x^0}}\,\Gamma_v$ commutes with $\Gamma_{\underline{x^2x^3}}$, eq. (3.39) is equivalent to:

$$\Gamma_{\underline{x^0}}\,\Gamma_v\,\epsilon\,\Big|_{\rho=0} = \epsilon\,\Big|_{\rho=0} \,. \tag{3.40}$$

Eq. (3.40) is the condition satisfied by the parameter of the supersymmetry preserved by a massless particle which moves in the direction of the vector $\mathbf{v}$ at $\rho = 0$, *i.e.* by a gravitational wave which propagates precisely along the trajectories found in section 2. However, the background projector $(i\sigma_2)\,\Gamma_{\underline{x^0\cdots x^3}}$ and the one of the probe, $\Gamma_{\underline{x^0}}\,\Gamma_v$ do not commute (actually, they anticommute). This means that the conditions (3.19) and (3.39) cannot be imposed at the same time and, therefore, the probe breaks completely the supersymmetry of the background. The most relevant aspect of this result is that the supersymmetry breaking produced by the probe is just identical to the one corresponding to a massless particle which moves in the direction of $\mathbf{v}$. This fact is a confirmation of our interpretation of the giant graviton configurations as blown up gravitons.

Notice that we have found the supersymmetry projection for the giant graviton configurations from the conditions (2.38) and (2.44). Clearly, we could have done our reasoning in reverse order and, instead, we could have imposed first that our brane probe breaks supersymmetry as a massless particle at $\rho = 0$. In this case we would arrive at the same conditions as those obtained by studying the hamiltonian and, actually, this would be an alternative way to derive them.

4 Summary and discussion

In this paper we have found configurations of a brane probe on the (D(p-2), Dp) background which behave as a massless particle. We have checked this fact by studying the motion of the brane and the way in which breaks supersymmetry. These giant graviton configurations admit the interpretation of a set of massless quanta polarized by the gauge fields of the background. Actually, by recalling the arguments of refs. [3]-[7], one can argue that there are two possible descriptions of this system, as a point-like particle or as an expanded brane, which are valid for different ranges of the momenta and cannot be simultaneously valid.

In the cases studied in refs. [3]-[7] the blow up of the gravitons takes place on a (fuzzy) sphere. In our case the brane probe shares two dimensions with the branes of the background and, thus, our gravitons are also expanded along a noncommutative plane. We have parametrized the volume occupied by the probe in the noncommutative plane by means of the flux of the worldvolume gauge field. If this flux is fixed and finite, the angular momentum of the brane is bounded for $p < 5$ and one realizes the stringy exclusion principle.

Let us finally comment on some possible extensions of our results. It is clear that one should investigate the spectrum of small vibrations of the brane around the giant graviton configuration, along the lines of refs. [8, 12], in order to determine its stability. It would be also very interesting to have a more explicit picture of the blow up of the gravitons in the noncommutative plane in terms of the Myers dielectric efect. Another topic which we would be interesting to examine is whether or not there exist configurations similar to the ones studied here in M-theory. Notice that the authors of the second paper in [14] found solutions of eleven dimensional supergravity generated by a non-threshold (M2,M5) bound state. The natural probe to consider in this case is a M5-brane sharing three common directions with the background. We expect to report on these issues in future.

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A. Supergravity analysis of the (D1,D3) background

In this appendix we shall study the supersymmetry preserved by the $(D1, D3)$ solution of the type IIB supergravity equations. In order to perform this analysis it is more convenient to work in the Einstein frame, in which the metric $ds^2_E = e^{-\phi_D/2}\, ds^2$ takes the form:

$$\begin{aligned} ds^2_E &= f_3^{-1/2}\, h_3^{-1/4} \Big[-(\,dx^0\,)^2 + (\,dx^1\,)^2 + h_3 \Big((\,dx^2\,)^2 + (\,dx^3\,)^2 \Big) \Big] + \\ &\quad + f_3^{1/2}\, h_3^{-1/4}\, dr^2 + f_3^{1/2}\, h_3^{-1/4}\, r^2 \Big[\frac{1}{1-\rho^2}\, d\rho^2 + (1-\rho^2) d\phi^2 + \rho^2 d\Omega_3^2 \Big] \,, \end{aligned} \tag{A.1}$$

where f_3 and h_3 are given in eq. (2.2) and, for simplicity, we have taken the string coupling constant g_s equal to one.

The supersymmetry transformations of the type IIB supergravity fields have been obtained in ref. [20]. In this reference the author uses complex spinors instead of working the real two-component spinor written in eq. (3.5). It is not difficult, however, to find the relation between these two notations. If ϵ_1 and ϵ_2 are the two components of the real spinor written in eq. (3.5), the complex spinor is simply:

$$\epsilon = \epsilon_1 + i\epsilon_2 \, . \tag{A.2}$$

From eqs. (3.5) and (A.2) it is straightforward to find the following rules to pass from one notation to the other:

$$\epsilon^* \leftrightarrow \sigma_3 \, \epsilon \, , \qquad i\epsilon^* \leftrightarrow \sigma_1 \, \epsilon \, , \qquad i\epsilon \leftrightarrow -i\sigma_2 \, \epsilon \, . \tag{A.3}$$

We will use the correspondence written in eq. (A.3) to express the result of our supergravity analysis in the notation (3.5).

With our notations for the gauge forms, the supersymmetry transformations of the dilatino λ and gravitino ψ in type IIB supergravity are [20]:

$$\begin{aligned}
\delta\lambda &= i\Gamma^M \epsilon^* \, P_M - \frac{1}{24} \, \Gamma^{M_1 M_2 M_3} \, \epsilon \, F_{M_1 M_2 M_3} \, , \\
\delta\psi_M &= D_M \epsilon - \frac{i}{1920} \, \Gamma^{M_1 \cdots M_5} \Gamma_M \, \epsilon \, F^{(5)}_{M_1 \cdots M_5} + \\
& \quad + \frac{1}{96} \left(\Gamma_M^{\;\; M_1 M_2 M_3} - 9 \, \delta_M^{M_1} \, \Gamma^{M_2 M_3} \right) \epsilon^* \, F_{M_1 M_2 M_3} \, .
\end{aligned} \tag{A.4}$$

In eq. (A.4) the Γ^M's are ten-dimensional Dirac matrices with curved indices, $F^{(5)}$ is the RR five-form and P_M and $F_{M_1 M_2 M_3}$ are given by:

$$\begin{aligned}
P_M &= \frac{1}{2} \left[\partial_M \phi_D + i e^{\phi_D} \, \partial_M \chi \right] \, , \\
F_{M_1 M_2 M_3} &= e^{-\frac{\phi_D}{2}} \, H_{M_1 M_2 M_3} + i e^{\frac{\phi_D}{2}} \, F^{(3)}_{M_1 M_2 M_3} \, ,
\end{aligned} \tag{A.5}$$

where χ is the RR scalar and H and $F^{(3)}$ are, respectively, the NSNS and RR three-form field strengths.

The solutions of the supergravity equations we are dealing with are purely bosonic and, thus, they are only invariant under those supersymmetry transformations which do not change the fermionic fields λ and ψ. Let us consider first the variation of the dilatino λ for the (D1,D3) background. From eqs. (2.1), (2.4) and (2.13) it follows that the non-vanishing components of the complex three-form F are:

$$\begin{aligned}
F_{01r} &= i \sin\varphi \, h_3^{\frac{1}{4}} \, \partial_r \, f_3^{-1} \, , \\
F_{23r} &= \sin\varphi \cos\varphi \, h_3^{\frac{7}{4}} \, \partial_r \, f_3^{-1} \, .
\end{aligned} \tag{A.6}$$

By using eq. (A.6), and by computing P_M from eq. (2.1), one easily finds from the first equation in (A.4) that the supersymmetry variation of λ is:

$$\delta\lambda = \frac{\sin\varphi}{4}\, f_3^{\frac{1}{2}}\, h_3^{\frac{5}{8}}\partial_r\, f_3^{-1}\,\Gamma_{\underline{r}}\Big[\, -i\sin\varphi\, h_3^{\frac{1}{2}}\, f_3^{-\frac{1}{2}}\epsilon^* \,+\, \Gamma_{\underline{x^0x^1}}\epsilon \,+\, i\cos\varphi\, h_3^{\frac{1}{2}}\,\Gamma_{\underline{x^2x^3}}\epsilon\,\Big]\,. \tag{A.7}$$

Clearly $\delta\lambda = 0$ if and only if the term inside the brackets on the right-hand side of eq. (A.7) vanishes. It is an easy exercise to verify, by using the correspondence (A.3), that the condition so obtained coincides with the κ-symmetry condition (3.11). Thus, we can represent the spinor ϵ as in eq. (3.13) with α given in eq. (3.12) and $\tilde\epsilon$ satisfying (3.14).

More information about the spinors which leave invariant the (D1,D3) solution can be gathered by looking at the gravitino transformation rule (A.4). Let us consider first the components of ψ along the directions x^μ ($\mu = 0,1,2,3$) parallel to the D3-brane. Due to the presence of a covariant derivative on the expression of $\delta\psi_M$, in order to obtain the variation of the gravitino, we need to know the value of the spin connection. It is straightforward to check that the only non-vanishing components of the latter of the type $\omega_{x^\mu}^{\underline{MN}}$ are:

$$\begin{aligned}
\omega_{x^0}^{\underline{x^0r}} &= \omega_{x^1}^{\underline{x^1r}} = \frac{1}{4}\, f_3^{\frac{1}{2}}\partial_r f_3^{-1} \,+\, \frac{1}{8}\,\sin^2\varphi\, h_3\, f_3^{-\frac{1}{2}}\partial_r f_3^{-1}\;,\\
\omega_{x^2}^{\underline{x^2r}} &= \omega_{x^3}^{\underline{x^3r}} = \frac{1}{4}\, f_3^{\frac{1}{2}}\, h_3^{\frac{1}{2}}\partial_r f_3^{-1} \,-\, \frac{3}{8}\,\sin^2\varphi\, h_3^{\frac{3}{2}}\, f_3^{-\frac{1}{2}}\partial_r f_3^{-1}\;.
\end{aligned} \tag{A.8}$$

By using eq. (A.8), the values of the forms given in eqs. (2.13) and (A.6) and the constraint imposed on ϵ by the invariance of the dilatino (eq. (3.11)), one concludes after some calculation that the gravitino components ψ_{x^μ} ($\mu = 0,1,2,3$) are invariant under supersymmetry if and only if the Killing spinor is independent of the x^μ's, namely:

$$\partial_{x^\mu}\,\epsilon \,=\, 0\;, \qquad (\mu = 0,1,2,3)\;. \tag{A.9}$$

The condition $\delta\psi_\rho = 0$ determines the dependence of ϵ on ρ. The relevant component of the spin connection needed in the calculation of $\delta\psi_\rho$ is:

$$\omega_\rho^{\underline{\rho r}} = \frac{1}{\sqrt{1-\rho^2}}\left[1 - \frac{1}{4}\, r\, f_3\,\partial_r\, f_3^{-1} \,+\, \frac{1}{8}\, r\,\sin^2\varphi\, h_3\,\partial_r\, f_3^{-1}\right]. \tag{A.10}$$

(Notice that when $r \to \infty$ we recover eq. (3.16)). Using again the condition imposed by the dilatino invariance, one can easily prove that the dependence on ρ of ϵ is the same as in eq. (3.17). Thus, we have verified the representation of ϵ in terms of the spinor $\hat\epsilon$ written in eq. (3.13) and (3.17). An explicit representation of $\hat\epsilon$ can be obtained by looking at the transformation of the other components of the gravitino. For instance, one can consider the equation for the radial component of ψ. Using the fact that $\omega_r^{\underline{MN}} = 0$, one gets that the dependence of $\hat\epsilon$ on r is determined by the equation:

$$\partial_r\hat\epsilon = -\frac{1}{8}\,\partial_r\Big[\,\ln\big(f_3 h_3^{\frac{1}{2}}\big)\Big]\;\hat\epsilon\;, \tag{A.11}$$

which can be immediately integrated. One can proceed similarly with the remaining components of ψ. The final result of this analysis is the complete determination of the form of

$\hat{\epsilon}$ and, therefore, of the complete Killing spinor ϵ. One gets:

$$\epsilon \;=\; \Big[\, f_3\, h_3^{1/2}\,\Big]^{-\frac{1}{8}}\; e^{\frac{1}{2}\,\alpha\,\Gamma_{\underline{x^2x^3}}\,\sigma_3}\; e^{-\frac{1}{2}\,\beta\,\Gamma_{\underline{\rho r}}}\; e^{-\frac{1}{2}\,\phi\,\Gamma_{\underline{\phi r}}}\;\times$$
$$\times\, e^{-\frac{1}{2}\,\theta_1\,\Gamma_{\underline{\theta_1\rho}}}\; e^{-\frac{1}{2}\,\theta_2\,\Gamma_{\underline{\theta_2\theta_1}}}\; e^{-\frac{1}{2}\,\theta_3\,\Gamma_{\underline{\theta_3\theta_2}}}\;\epsilon_0\;, \tag{A.12}$$

where ϵ_0 is a constant spinor satisfying the condition

$$(i\sigma_2)\Gamma_{\underline{x^0\cdots x^3}}\,\epsilon_0 \;=\; \epsilon_0\;. \tag{A.13}$$

It follows from eqs. (A.12) and (A.13) that the (D1,D3) background is $\frac{1}{2}$ supersymmetric.

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